

Preface.....	7
Introduction.....	10
Chapter 1. General convergence theorems of functions of statistics.....	24
1.1 Statistics as functionals of empirical distributions.....	24
1.2 The functional types: integral, conditional, characterizing, augmented functionals	25
1.3 Estimability of functionals. Functionals with peculiarities.....	32
1.4 Parametric and non-parametric approaches to the estimation of statistical characteristics.....	34
1.5 Non-parametric estimators of kernel type.....	36
1.6 Asymptotic characteristics of statistics	38
1.7 Convergence in law of the first moments of the functions of statistics	41
1.7.1 The simplest case	41
1.7.2 Convergence to standard distributions.....	42
1.8 Convergence in mean square of the substitution estimator	43
1.9 Deviation moments of the substitution estimator, its MSE, bias and variance.....	46
1.9.1 The simplest case (deviation moments).....	46
1.9.2 The simplest case (MSE, bias and variance)	49
1.9.3 Case when the function $H(t)$ depends on n	53
1.9.4 Case of a stationary point t of the function $H(t)$	54
1.9.5 Mixed deviation moments of the substitution estimator.....	56
1.10 Piecewise smooth approximation of the substitution estimator and its convergence in square mean	58
1.11 Deviation moments of a piecewise smooth approximation of the substitution estimator, its MSE.....	62
1.12 Optimization problem for a piecewise smooth approximation of the substitution estimator.....	65
1.13 Convergence in distribution of a piecewise smooth approximation of the substitution estimator	67
1.14 Simple examples: the case of parametric estimation	68
Chapter 2. Kernel estimators of functionals from independent samples	71
2.1 Auxiliary results for kernel estimators of basic functionals of basic functionals and their derivatives	71
2.2 Convergence in mean square of estimators of the basic functionals and their derivatives	79
2.2.1 Convergence of the covariance matrix	79
2.2.2 The bias convergence.....	81
2.2.3 The improvement of the rate of convergence of MSE.....	84
2.2.4 Convergence of estimators of derivatives of the basic functionals.....	88
2.3 Convergence in distribution of the estimators of the basic functionals and their derivatives	92
2.4 Convergence of functions of the estimators of the basic functionals	96
2.5 Asymptotically optimal estimator of a multivariate density distribution and its derivatives	98
2.5.1 Problem setting	101
2.5.2 Convergence in mean square of kernel estimators	108
2.5.3 Construction of optimal kernel estimators.....	118
2.5.4 Asymptotic normality of kernel estimators.....	124
2.5.5 Convergence with probability one.....	129
2.6 Convergence of estimators of characterizing functionals.....	131

2.6.1 Weak convergence	131
2.6.2 Pointwise convergence in quadratic mean.....	132
2.6.3 Uniform convergence in mean square	134
2.6.4 Joint asymptotic distribution of the estimators of the characterizing functionals.....	136
2.7 Examples of estimating specific functionals.....	139
2.7.1 Estimation of the product of densities	140
2.7.2 Kernel estimator of regression function.....	141
Chapter 3. Estimation of functional from depend samples	148
3.1 Conditions of weak dependence	148
3.1.1 The notion of weak dependence	148
3.1.2 Relations between different coefficients of weak dependence	149
3.1.3 Examples of sequences with mixing.....	150
3.1.4 Strong mixing conditions for functions of stationary processes.....	151
3.1.5 S.m. conditions for dynamic observation models.....	154
3.1.6 Properties of a sample with overlapping.....	163
3.1.7 Some other forms of weak dependence	164
3.2 Estimation of the logarithmic derivative of a density with the use of piecewise smooth approximations from sequences with s.m	166
3.2.1 Statement of the problem	166
3.2.2 Kernel estimators of a distribution density and its derivative from an independent sample.....	167
3.2.3 Properties of the kernel density estimator from sequences with s.m	170
3.2.4 Properties of the kernel estimator of the density derivative for the s.m. sequences	172
3.2.5 Convergence of the fourth moments of the kernel estimator of a density and its derivative for the s.m. sequence	173
3.2.6 Properties of the kernel estimator of the logarithmic derivative of a distribution density for s.m. sequences	174
3.3 Proof of the results of Section 3.2.....	177
3.4 Estimation of the logarithmic gradient of a density from s.m. vector sequences	193
3.5 Estimation of conditional functionals from s.m. sequences	199
3.5.1 Statement of the problem	199
3.5.2 Piecewise smooth approximation of classical estimators of Nadaraya–Watson type.....	201
3.5.3 Kernel estimators of the basic functionals of an independent sample	202
3.5.4 The properties of kernel estimators of the basic functionals from s.m. sequences	206
3.5.5 Convergence of the fourth moments of kernel estimators for the basic functionals from s.m. sequences	208
3.5.6 The MSE of the substitution estimators $J_n(x)$ and their piecewise smooth approximation	209
3.5.7 Convergence of the covariance matrices of the estimators of the basic functionals.....	211
3.5.8 Properties of the substitution estimator of the conditional functional and its piecewise smooth approximation for s.m. sequences	212
3.5.9 Non-parametric identification of a nonlinear autoregression	216
3.6 Proof of the results of Section 3.52.....	18
Chapter 4. Non-parametric estimation of functionals of distributions from sequences of	

observations with additive dependent noises	233
4.1 Non-parametric estimation of derivatives of a multivariate distribution density	233
4.1.1 Introductory notes	234
4.1.2 Statement of the problem	235
4.1.3 Convergence in the quadratic mean	238
4.1.4 Optimization of the MSE convergence rate	252
4.1.5 Asymptotic normality	253
4.1.6 Convergence with probability one	256
4.2 Non-parametric estimation of functions of a distribution density	258
4.2.1 State of the problem	258
4.2.2 The setting for the problem	259
4.2.3 Basic assumptions and definitions	260
4.2.4 Asymptotic properties of \hat{f}	262
4.2.5 Estimation of $V(f_*)$	266
4.3 Non-parametric estimation of the ratios of derivatives of a multivariate distribution density	268
4.3.1 Statement of the problem	269
4.3.2 Asymptotic properties $\hat{f}_a^{(\alpha)}(t)$	270
4.3.3 Estimation of the ratio $\Psi(t)$	272
4.4 Non-parametric estimation of linear functionals of a distribution function	273
4.4.1 Statement of the problem	273
4.4.2 Optimization of the convergence rate of the estimators of functionals	274
4.4.3 Optimization of the MSE of the estimators	278
Chapter 5. Construction of probabilistic models of dynamic systems	283
5.1 Construction of probabilistic models of dynamical systems	283
5.1.1 Statement of the problem	283
5.1.2 Deterministic regression	288
5.1.3 Appendix	293
5.1.4 Stochastic regression	295
5.1.5 Autoregression of the first order	305
5.1.6 Bivariate autoregression process	309
5.1.7 Autoregression of order p	312
5.1.8 Autoregression with a control	320
5.2 Guaranteed inference on the estimation of noise characteristics in linear regressions	326
5.2.1 Sequential estimation of the noise distribution law in the scheme of random regression	327
5.2.2 Sequential estimation of a density and its derivatives	336
5.2.3 Sequential estimation of linear functionals	341
5.2.4 Sequential goodness-of-fit test for a composite hypothesis about an unknown distribution	346
5.2.5 Sequential estimation of noise distributions for dynamical systems with stationary inputs	350
Chapter 6. Non-parametric filtration of random sequences	354
6.1 General problems of signal processing	354
6.2 Statement of a filtering problem	355
6.2.1 Criteria	355

6.2.2 Approximate solution methods	357
6.2.3 Empirical Bayesian approach	358
6.2.4 Asymptotically optimal procedures	359
6.3 Conditions of the asymptotic optimality of estimators	360
6.3.1 Necessary and sufficient conditions of the asymptotic optimality	361
6.3.2 Sufficient condition of asymptotic optimality	362
6.4 Static observation models	364
6.4.1 Loss function.....	365
6.5 Transformation formula for posterior probabilities	371
6.6 Optimal filtering equation for static models	373
6.7 Non-parametric version of the optimal filtering equation for static models.....	376
6.8 Dynamic observation models.....	379
6.8.1 Conditionally-exponential family	379
6.8.2 Loss function.....	382
6.9 Optimal filtering equation for dynamic models.....	383
6.10 Filtering of some functions of a useful signal.....	393
6.11 Asymptotically ϵ -optimal filtering procedure.....	401
6.11.1 Density estimation from one realization of the process.....	401
6.11.2 Selection criterion for the length of dependence zone.....	402
6.11.3 Kalman's filter and the asymptotically ϵ -optimal estimator	405
6.11.4 The choice of the dependence zone for finite n	408
6.11.5 Length of the dependence zone by unknown signal characteristics	413
Chapter 7. Non-parametric interpolation and prediction of random signals	417
7.1 The equation for optimal nonlinear interpolation	417
7.2 Non-parametric counterpart of the interpolating equation.....	423
7.3 Prediction of an observable stationary sequence	427
7.4 Prediction of an unobservable component of a partly observable Markov sequence... <td>429</td>	429
7.5 Examples of the problems of predicting	433
7.6 Quality of estimation in problems of signal processing.....	438
7.7 Empirical risk formula for the problem of filtering	439
7.8 The empirical risk in problems of interpolation	442
7.9 Risk estimation in problems of prediction	444
7.10 Convergence of empirical estimators of risks.....	446
7.11 Examples of non-parametric risk estimators.....	449
7.12 Algorithm for the simultaneous choice of the realization length and the degree of dependence of an observable process	451
Chapter 8. Non-parametric selection methods of stepwise Markov processes	457
8.1 The model of observations and the condition of weak dependence	457
8.2 Non-parametric filtration of finite-dimensional Markov chains.....	458
8.3 Non-parametric interpolation of finite-dimensional Markov chains	462
8.4 Estimating the times of changes in the properties of random processes.....	466
References	472